

Identifying the Contributors to World Pollution

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1 Introduction

The field of pollution economics¹ has received widespread theoretical and empirical attention of late. Forecasts by climate scientists concerning the ecological dangers of accelerated levels of greenhouse gas emissions have become increasingly dire, and studies estimating the potential economic impact of unbridled climate change have been similarly dire in tow. According to the pre-eminent “Stern Review on the Economics of Climate Change” (2006), adopting a “business-as-usual” approach to the world’s current and future emissions activities would likely cause a decrease in the world’s present value discounted level of future welfare by the equivalent of a reduction in per capita consumption of between 5 and 20 percent. This deprivation will be caused by, among other things, increased flood risks, water shortages, declining agricultural and wild fishery yields, increases in deaths from exposure to extreme temperatures and natural disasters, greater incidences of vector-borne diseases such as malaria and dengue fever, and species extinction. All things considered, the importance of developing a theory for understanding how best to reduce the world’s greenhouse gas emissions is vital.

Any system of internationally unregulated greenhouse gas emissions will generate market failures of at least three sorts. The first derives from the status of greenhouse gases as “global” pollutants. Since greenhouse gases diffuse rapidly upon entering the atmosphere, wielding their environmental influence remotely rather than locally, the social costs of a given polluter’s emissions are bound to be quite geographically widespread. Absent regulations to counteract that externality, polluters do not fully account for the costs of their emissions and therefore overpollute relative to the socially efficient amount.

Models designed to capture the phenomenon of global pollution will typically treat it as a universal public bad, whereby agents’ emissions decisions harm not only their fellow countrymen, but the rest of the world as well. The universality of the accompanying free-rider effect has significant consequences for policy. Since domestic governments are rarely wont to weigh the interests of other countries as heavily as their own, unilateral or non-cooperative government interventions are unlikely to internalize the costs of a country’s emissions. International regulators, however, would be theoretically poised to do just that.

The second market failure caused by unregulated emissions results from the time lag that exists between the moment that emissions occur and the horizon over which its costs are realized. When such “lag effects” exist, current generations are able to enrich themselves at the expense of future ones by degrading the environment beyond the socially optimal level. Any society that politically or economically disenfranchises the young exacerbates

¹Or “Pollutionomics,” as the prevailing formula for pop-economic nomenclature would have it.

the lag effect by fostering a government that is not properly attuned to the interests of everyone. Consequently, those societies may fail to develop policies sufficient to control for the intergenerational externalities created by pollution.

Models that are meant to capture the lag effects from pollution must be dynamic, but they must also probably include explicit socio-political assumptions about how a government will respond to the needs of its citizens. Insofar as future generations are unable to represent their interests effectively to regulators (having in most cases not even been born yet!), it is difficult to see how a government tasked with representing the interests of its voters may be persuaded to correct for intergenerational externalities at any level of jurisdiction.

The final type of market failure is distributional in nature. Even though rich countries account for the vast majority of emissions activities, poor countries are the ones that are predicted to be the most hurt by them. This likely exacerbates the externalities created by relatively rich countries, which can free-ride on the abatement efforts of their poorer neighbors. Concomitantly, however, it may decrease the externalities occasioned by the countries that bear the greatest disutilities from pollution. These contrary effects indicate that the overall effect of international inequality on worldwide pollution may actually be ambiguous. Nonetheless, it does create economic distortions, and it does inspire questions about fairness. Should the poorest countries in the world really be the one's expected to foot the bill for first-world pollution? In an ideal world, one would think not.

Models that wish to capture distributional market failures must obviously include some degree of international heterogeneity. In particular, evidence suggests, those models should assume that countries can suffer from pollution in differing amounts merely by virtue of their geographical locations. Furthermore, they should address how it is that rich countries can account for more overall emissions than poor ones.

The present paper focuses on just two of the market failures characterized above, the so-called "static" ones. By mimicking the general equilibrium production and trade model outlined by Brian Copeland and Scott Taylor in their 2001 article "International Trade and the Environment: A Framework for Analysis," the model analyzes the equilibrium that result from treating pollution as a universal public bad whose effects are unequally felt. As in the foundational model, pollution is treated as a bi-product of goods production and enters directly into the utility functions of households. However, it does so in a different way; instead of countries experiencing only their own pollution, they are assumed to suffer from the total amount of pollution created worldwide.

The globalization of pollution changes the countries' optimization problems substantially. When each government's citizens are affected by the emissions levels of the other countries, the government will try to maximize national welfare given what it expect the other countries

to pollute. To solve for the equilibrium level of pollution in this environment, my model invokes the concept of Nash equilibrium, whereby each country responds to the actions of the others in a manner most beneficial to its own interests.

2 The Model

Imagine a world consisting of $n \in \mathbb{N}$ countries, each of which is the formal equivalent of an open economy composed of utility-maximizing consumers, profit-maximizing firms, and a surplus-maximizing national government.

2.1 Consumers

The consumers born within a country are completely homogeneous. Relative to each other they are endowed with the same opportunities, express the same desires, possess the same abilities, and have perfect information about the surrounding domestic economy. They differ from their international counterparts in only two respects: their endowments of capital and labor (captured by the variables K_i and L_i , respectively) and the extent to which they suffer from the effects of aggregate pollution are unique for each country. Since each population is homogeneous, it can be represented in the aggregate by a representative consumer whose qualities are proportional to those of every individual. Normalizing the population size of the country to 1, the utility of country i 's representative consumer can be defined as

$$U_i \equiv u(x_i, y_i) - h_i(Z), \quad (1)$$

where x_i and y_i are the consumer's consumption levels of goods X and Y and

$$Z \equiv \sum_{j=1}^n Z_j \quad (2)$$

is the aggregate level of world pollution. Copeland and Taylor (2001) assume that u is continuously differentiable and strictly concave along its arguments, but for the purposes of the present paper it will help to assume that u exhibits the Cobb-Douglas form $u(x_i, y_i) \equiv (x_i)^\beta (y_i)^{1-\beta}$, where $\beta \in (0, 1)$, which shares those characteristics.

The function h_i represents the extent to which the people of country i are hurt by aggregate pollution. To add more structure to the model, let it be defined as

$$h_i(Z) \equiv \frac{Z^{\gamma_i}}{\gamma_i}, \quad (3)$$

where $\gamma_i \geq 1$ to ensure that the marginal disutility from pollution is nondecreasing.² One should note that since the disutility from consuming Z is assumed to be additively separable from the utility from consuming x_i and y_i , the consumers' preferences over private consumption are independent of the level of Z . In other words, a change in pollution has no effect on the marginal rate of substitution between x_i and y_i .

2.2 Firms

Every country contains a multitude of firms capable of producing goods X and Y . The firms within each country are identical, but they differ from country to country in exactly two respects: since labor and capital are by assumption perfectly immobile, each country contains its own set of factor prices; furthermore, each country has access to its own abatement technology, which is captured by the parameter $\alpha_i \in (0, 1)$. As the production technologies of section 4.1 of the Appendix show, the higher the abatement parameter, the higher the country's emissions will be given any choice of the firm's inputs and abatement efforts, that is to say, the less efficiently the firm will be able to reduce its emissions.

Firms choose their levels of capital and labor for each good to maximize profits. They take goods prices, factor prices, and the price of pollution as given. Since industry entry is assumed to be free and the firms within each country are identical, no firm makes positive profits. An important implication of this is that, collectively, the firms will act to maximize national income. Another is that their activities can be aggregated into the actions of just one firm without losing any of their generality. From now on I will therefore treat each country as if it contains one representative firm producing both goods.

Firms create pollution as a bi-product of producing good X . Copeland and Taylor (2001) demonstrate that when this is the case (and a few functional form assumptions are made), the goods' production functions can be manipulated to allow pollution to become an input instead³. The interpretation of Z_i then translates into that of the level of "environmental services" called upon in the act of production. Beginning with the functions that result from this transformation, I restrict the production technology even further and assume that it occurs in country i according to the equations

$$Y_i = F(K_i^x, L_i^x); \tag{4}$$

$$X_i = (Z_i)^{\alpha_i} [F(K_i^y, L_i^y)]^{1-\alpha_i}, \tag{5}$$

where $F(K, L) \equiv K^\delta L^{1-\delta}$ and $\delta \in (0, 1)$. Just like the country's utility function, its produc-

²This functional form is borrowed from Copeland and Taylor (1995).

³I demonstrate the conversion of pollution from a bi-product to an input in section 1 of the Appendix

tion functions exhibit the Cobb-Douglas form. The superscripts located above the arguments of F indicate the good toward which the input is dedicated. Assuming that both goods are fully employed, one requires that for every i ,

$$K_i^x + K_i^y = K_i \quad (6)$$

and

$$L_i^x + L_i^y = L_i. \quad (7)$$

An underlying condition of the transformed production technologies is that

$$X_i \geq Z_i, \quad (8)$$

which results from the specific way that the fundamental technologies must be designed to allow for the translation of Z_i from bi-product to input.⁴

2.3 Governments

The strategic behavior of domestic governments is the primary subject of study. Every government is the formal equivalent of a rational social planner representing the interests of its people perfectly. It has one instrument at its disposal: through either taxation or a permit system, it has unilateral control over the level of domestic pollution. Since firms are perfectly competitive, it can be shown that both systems produce the same outcome. For convenience, therefore, I will assume that the preferred regulatory system is a permit system wherein the government chooses the country's optimal level of pollution (denoted by Z_i) and then firms bid up its price to the point that their profits are zero.

Recalling that the separability of the utility function implied that the consumers' maximization problem did not rely on the level of pollution, the government can be thought of as choosing the consumption variables x_i and y_i on their behalf without affecting the results. Imposing good Y as the numeraire and taking p and $Z_{-i} \equiv \sum_{j \neq i} Z_j$ as given, the government's objective is to

$$\max_{x_i, y_i, 0 \leq Z_i \leq X_i} \{u(x_i, y_i) - h_i(Z_i + Z_{-i}); px_i + y_i = I_i\}, \quad (9)$$

where I_i is the national income function corresponding to the production technologies delin-

⁴See section 1 of the Appendix for more on this.

eated above. Formally, it may be defined as

$$I_i = I(p, \alpha_i, K_i, L_i, Z_i) \equiv \max_{X_i, Y_i} \{pX_i + Y_i; (X_i, Y_i) \in T(\alpha_i, K_i, L_i, Z_i)\}, \quad (10)$$

where T is the country's production possibilities correspondence defined by equations (4)-(8).

Assuming that $X_i \geq Z_i$ (8) is not binding, the solution to the income maximization problem is

$$I_i = F(K_i, L_i) + \tilde{a}_i(p)Z_i, \quad (11)$$

where $\tilde{a}_i(p) \equiv \left(\frac{\alpha_i}{1-\alpha_i}\right) [p(1-\alpha_i)]^{1/\alpha_i}$.⁵

If it binds, however, then national income will be somewhat lower. Using the result from section 4.2 that $X_i = Z_i [p(1-\alpha_i)]^{\frac{1-\alpha_i}{\alpha_i}}$ when condition (8) is ignored, it is clear that the condition will only bind when $\alpha_i > \frac{p-1}{p}$. In that case, given Z_i , the firm's best option is to produce at a point where X_i is equal to it.⁶ Using the production technology for X_i given in equation (5), the firm must therefore choose K_i^x and L_i^x to satisfy

$$F(K_i^x, L_i^x) = Z_i. \quad (12)$$

Given this simplification, the new income maximization problem is to choose Y_i , K_i^x , K_i^y , L_i^x , and L_i^y to maximize $pZ_i + Y_i$ subject to production technology for Y_i (equation (4)), the full employment conditions on K_i and L_i (equations (6) and (7)), and the new requirement of equation (12). When $X_i \geq Z_i$ binds, therefore, the maximum value of national income is

$$I_i = F(K_i, L_i) + (p-1)Z_i. \quad (13)$$

Figure 1 illustrates how national income is maximized in the two cases.⁸ Given some chosen level of pollution, the country's production possibilities frontier (PPF) well-defined. At that point, if one ignores condition (8), then the optimal level of production, which has been labelled as (X^*, Y^*) on the left-hand graph, will occur at the point along the PPF where $\frac{dY}{dX} = -p$. Since Y is the numeraire good, the solution for I , denoted by I^* , will be the intercept of the iso-income function tangent to the point of production. However, if the particular level of Z required to generate the PPF is so large that $\frac{dY}{dX} = -p$ would occur at some point where $X^* < Z$, as is shown in the right-hand graph, then I^* will not be attainable. The best that the firm can do, then, is to shift its production level down

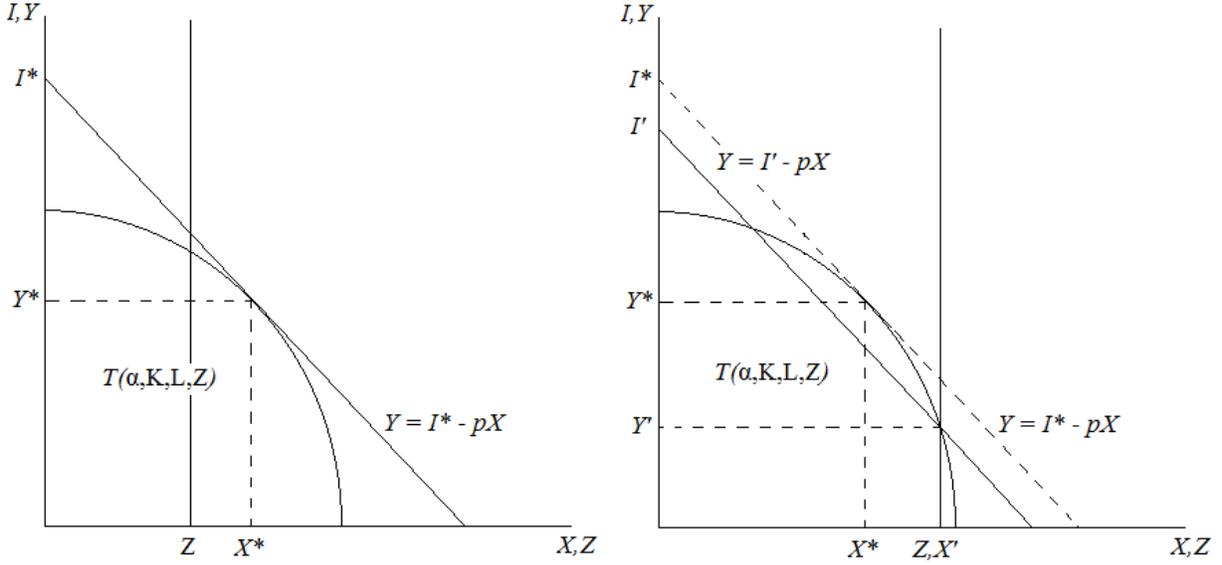
⁵This is shown in section 4.2.

⁶This relies on the easily verifiable fact that given any parameter profile, $T(\cdot)$ is a strictly convex set.

⁷Again, for a proof see section 4.2 in the Appendix

⁸All of the variables' subscripts in the figure have been suppressed, so I suppress them in the proceeding discussion as well.

Figure 1: Solving for National Income.



and along the frontier to the point where $X = Z$. As before, the country's income will be determined by the iso-income line that intersects with the chosen point. Production will occur at the point $(X', Y') = (Z, Y')$, while national income will be given by $I' = Y' + pZ$.

Combining equations (11) and (13), the maximum level of national income is defined by the following piecewise function:

$$I_i = \begin{cases} F(K_i, L_i) + \tilde{a}_i(p)Z_i & \text{if } \alpha_i \leq \frac{p-1}{p}; \\ F(K_i, L_i) + (p-1)Z_i & \text{otherwise.} \end{cases} \quad (14)$$

Note that in both cases, the solution for I_i manages to separate the exogenous variables K_i and L_i from Z_i , α_i , and p . This is due to the assumption that the goods X and Y should share the function F as a part of their production technologies. What it ultimately entails is that K_i and L_i will fail to exert any impact on the government's optimal choice of Z_i .

It is easy to see now how a country could enrich itself by polluting. In general, increasing Z_i expands the firm's PPF by a permitting a greater level of X_i for every given level of Y_i . If α_i is less than $\frac{p-1}{p}$, then this will increase the country's income by allowing the new iso-income line to occur tangent to the PPF at a higher level of X_i . However, if $\alpha_i > \frac{p-1}{p}$, then $X_i = Z_i$ is the income-maximizing solution for X_i , and an increase of Z_i by Δ_z will lead to an equal increase in X_i . The new revenue garnered by such a change would therefore be $p \cdot \Delta_z$. However, since $Y_i = F(K_i, L_i) - Z_i$, there would be a corresponding loss of revenue from the adjustment in the amount of Δ_z . The net change in revenue from increasing Z_i by

Δ_z when $\alpha_i > \frac{p-1}{p}$ is therefore equal to $\Delta_z \cdot (p-1)$. If $p > 1$, this will lead to an increase in national income. But if $p < 1$, in which case α_i is always greater than $\frac{p-1}{p}$, then increasing Z_i will actually have an immiserating effect on national income by drawing capital and labor toward the production of the relatively low-revenue good X so that the condition $X_i \geq Z_i$ remains satisfied. In that case, surprisingly, it would be best from the point of view of maximizing national income not to pollute at all. As I will show, however, $p < 1$ will never occur in equilibrium.

Employing the expression in equation (14), the derivative of national income with respect to the country's pollution level is

$$I_{i,z} = \begin{cases} \tilde{a}_i(p) & \text{if } \alpha_i \leq \frac{p-1}{p}; \\ (p-1) & \text{otherwise.} \end{cases} \quad (15)$$

Since $\alpha_i = \frac{p-1}{p}$ implies that $\tilde{a}_i(p) = p-1$, $I_{i,z}$ is a continuous function of p . It is trivially a continuous function of α_i as well.

Returning to the government's primary problem as stated in expression (9), and defining λ_i as the multiplier of the problem's associated Lagrangian function, one can derive the following first order conditions confronting the government:

[x_i]

$$u_x(x_i^*, y_i^*) = \lambda_i^* p;$$

[y_i]

$$u_y(x_i^*, y_i^*) = \lambda_i^*;$$

[λ_i]

$$px_i^* + y_i^* = I_i(\cdot, Z_i^*);$$

[Z_i]

$$h_i'(Z_i^* + Z_{-i}) = \lambda_i^* I_{i,z}(\cdot, Z_i^*). \quad (16)$$

Notice that the first three equations are the same as they would be if the consumer were maximizing its utility given Z_i^* , reinforcing the point that there is no difference between the consumer choosing x_i and y_i and the government doing it. The special convenience furnished by assuming that u exhibits a Cobb-Douglas form now becomes apparent. Given any p , the

solution for the shadow price λ_i^* is the following constant:

$$\lambda_i^* = \lambda^*(p) \equiv \left(\frac{\beta}{p}\right)^\beta (1 - \beta)^{1-\beta}.^9 \quad (17)$$

λ_i^* does not depend on I_i , nor likewise on i , which has convenient consequences later on for determining which countries are going to pollute before the others. The interpretation of the shadow price in this context is that it is the differential gain in utility that results from increasing national income by one unit of Y .

Equation (16) is the bread and butter of the government's pollution optimization problem. In one simple equation, it embodies the maximizing behavior of a country's firms, households, and government. Since $I_{i,z}$ is piecewise, it yields a piecewise solution for Z_i^* given p and Z_{-i} of

$$Z_i^*(p, Z_{-i}; \beta, \gamma_i, \alpha_i) = \begin{cases} [\lambda^*(p)\tilde{a}_i(p)]^{\frac{1}{\gamma_i-1}} - Z_{-i} & \text{if } \alpha_i \leq \frac{p-1}{p}; \\ [\lambda^*(p)(p-1)]^{\frac{1}{\gamma_i-1}} - Z_{-i} & \text{otherwise.} \end{cases} \quad (18)$$

Up to this point, I have been ignoring the constraint that $Z_i \geq 0$, which states that the country is not allowed to negatively pollute, as well as condition (8). Dividing the production function for X_i on both sides by Z_i , maximizing $F(K_i^x, L_i^x)$ by choosing $K_i^x = K_i$ and $L_i^x = L_i$, and imposing condition (8) reveals that the maximum value of Z_i that any one country can produce is equal to $F(K_i, L_i)$. Imposing these further constraints on Z_i provides a final solution for the government's supply of pollution, to be denoted by \hat{Z}_i , given p and Z_{-i} , as

$$\hat{Z}_i(p, Z_{-i}) = \max \left\{ 0, \min \{ Z_i^*(p, Z_{-i}), F(K_i, L_i) \} \right\}. \quad (19)$$

The process for the government to compute its best response to Z_{-i} given p is first to compare α_i and $\frac{p-1}{p}$ to determine the relevant value of $I_{i,z}$ from equation (15); next is to calculate the optimal level of Z_i using equations (18) and (19); the final step is to issue the desired amount of permits, letting domestic firms bid on them until the point that their profits are zero. In the case in which the government chooses $Z_i = F(K_i, L_i)$, firms will be operating in the realm in which $X_i = Z_i$ even though it may be possible that $\alpha_i \leq \frac{p-1}{p}$. In such circumstances, the relevant value for $I_{i,z}$ will still have been determined by the relationship between α_i and p . The reason for this is that $Z_i = F(K_i, L_i)$ occurs on the rightmost border of the PPF where the derivative is undefined. As the border is approached, however, the usual constrained maximization calculus would still apply.

⁹To see this, recall that when u is Cobb-Douglas, $x_i^*(p, I_i) = \frac{\beta I_i}{p}$ and $y_i^* = (1 - \beta)I_i$, since Y is the numeraire. Plugging these into $u_y(x_i, y_i) = (1 - \beta)\left(\frac{x_i}{y_i}\right)^\beta$ and then recognizing from the first order conditions that $u_y(x_i^*, y_i^*) = \lambda_i^*$ yields the desired result.

The same goes for any country whose optimal pollution supply is 0. To avoid polluting requires that a country produce none of the pollution-intensive good. Instead, it must dedicate all of its production to good Y . As I will discuss later, the countries that desire the lowest levels of pollution will generally specialize in production of the clean good.

2.4 Nash Equilibrium

Now that the best response function of each government has been determined, it is possible to define a Nash equilibrium:

Definition 1. *Given some relative price p for the pollution intensive good X , a **Nash Equilibrium** is a profile $(\hat{Z}_1, \dots, \hat{Z}_n)$ of government strategies such that for every $i \in \{1, \dots, n\}$,*

$$\hat{Z}_i = \max \left\{ 0, \min \{ Z_i^*(p, Z_{-i}), F(K_i, L_i) \} \right\}, \quad (20)$$

where $\hat{Z}_{-i} \equiv \sum_{j \neq i} \hat{Z}_j(p, Z_{-j})$ and $Z_i^*(p, Z_{-i})$ is as defined by equation (18).

Given any p , it is difficult neither to find the Nash equilibrium level of Z nor to identify who contributes to world pollution. But like Z , p is endogenous, and it depends in part on the amount and distribution of world pollution. To disentangle the two variables, it would therefore be necessary to find an expression for p in terms of the system's exogenous variables and then plug it into the pollution demand equations to determine which countries have the highest demands for Z . But when α_i and γ_i are allowed to arbitrarily vary from country to country, such closed-form solutions are precluded. Instead, I will make an effort to narrow down the most suspect polluters by way of some comparative statics.

Before proceeding any further, however, I must state the country's demand for pollution. Denoting the set of contributing countries by C and its cardinality by c , country i 's demand for world pollution given p is defined by¹⁰

$$D_i(p) \equiv \begin{cases} [\lambda^*(p) \tilde{a}_i(p)]^{\frac{1}{\gamma_i-1}} & \text{if } \alpha_i \leq \frac{p-1}{p}; \\ [\lambda^*(p)(p-1)]^{\frac{1}{\gamma_i-1}} & \text{otherwise.} \end{cases} \quad (21)$$

Following Andreoni (1993), since $D_i(p)$ does not depend on Z_{-i} , it may also be described as the “free-rider inducing level of supply of Z_{-i} .” That is to say, if the other countries contribute at least $D_i(p)$, then country i 's best response will be to contribute nothing.

The first claim concerns the general equilibrium value of p . Due to the assumptions on the production technologies, and because pollution creates a direct disutility for every

¹⁰This follows immediately from equation (18).

country including the one doing the polluting, the price of the pollution-intensive good will always exceed that of the clean good. Thus, we have:

Claim 1. $p > 1$ in equilibrium.

Proof. Suppose not. Then $\alpha_i > \frac{p-1}{p}$ for every $\alpha_i \in (0, 1)$, so that for all i , country i 's relevant demand for pollution is

$$D_i(p) = [\lambda^*(p)(p-1)]^{\frac{1}{(\gamma_i-1)}},$$

which would be less than or equal to 0 since $p \leq 0$. Therefore, total pollution will be naught, which can only be achieved if $X_i = 0 \forall i$. Thus, $x_i = 0 \forall i$ will result as well, which would never occur because $\lim_{x \rightarrow 0} u_1(x, y) = \infty$, indicating that the marginal willingness to pay for good X at $x_i = 0$ is infinite. As such, any country's firms would find it profitable to produce and sell X . \square

The second claim begins to codify the relationship between the countries' demand equations and the set of contributors by demonstrating that the country with the highest demand will always contribute, as long as it is unique.

Claim 2. *Given any $p > 1$, the country with the highest demand, assuming that it is unique, will be a contributor.*

Proof. (An analogue to that of Bergstrom, Blume, and Varian (1986).) The first point to notice is that no country ever has a best response to add to world pollution if its demand has already been (weakly) exceeded. Therefore, total pollution should never exceed the demand of the highest demanding country. If it were to, then each contributor's best response would be to decrease its level of pollution, thus moving closer to its desired level. Nor, similarly, is it ever optimal for a given contingent of countries to contribute more to pollution than their highest-demanding member demands. Thus, the most that the countries beneath the highest demanding country in the world would ever be willing to pollute is equal to the demand of the next highest demanding country. As such, the country with the highest demand will have a best response to satisfy its own demand by filling the gap between what the other countries are willing to supply and what it itself demands. \square

What this claim suggests is that equilibria may exist wherein the country with the highest demand for pollution specializes in producing the pollution-intensive good. However, due to the upper limit on what it can pollute, even a country that specializes in producing X may not be sufficient to satisfy the demands of the other countries. If that is the case, then the next contributor will be the country with the next highest demand. If that contributor specializes in X as well, and the demands of remaining countries still have not been met, then

more and more countries will become contributors until a point is reached where $Z \geq D_i(p)$ for the non-contributor i with the highest demand. Since non-contributors by definition contribute nothing, they will simultaneously all specialize in producing the clean good.

The third claim demonstrates the relationship between a country's abatement technology and its demand for pollution. Because those countries with the largest demands are the one's who pollute, it indicates which countries might bear the greatest burdens of pollution based on their underlying parameters.

Claim 3. For any $\gamma_i > 1$ and $p > 1$, as $\alpha_i \rightarrow 0$, $D_i(p) \rightarrow \infty$.

Proof. As α_i drops, the country will achieve $\alpha_i < \frac{p-1}{p}$. Therefore, relevant demand is

$$D_i(p) = [\lambda^*(p)\tilde{a}_i(p)]^{\frac{1}{(\gamma_i-1)}}.$$

As $\alpha_i \rightarrow 0$, $\tilde{a}_i(p) \rightarrow \infty$. Therefore, so does $D_i(p)$. □

Ignorant of the γ parameters, the countries that are the most efficient at abating are more likely to specialize in the pollution-intensive good. There is no similar result for the γ 's, whose impact on pollution demand depends more intimately on the equilibrium value of p . It is easy to see, however, how the countries' relative disutilities from pollution might interfere with the positive result of the claim. If a country is relatively inefficient at abating (α_i high) but does not mind pollution very much (γ_i close to 1), that country may still have a great demand for pollution. It may specialize in the pollution-intensive good even as other countries that are better at abating remain contently committed to producing the clean good. As of right now, the countries are not permitted to pay each other to increase or reduce their pollution levels. As such, the differential preferences that the countries exhibit toward pollution are capable of generating a market failure beyond that of the stereotypical free-rider effect.

3 Conclusion

In this paper, I have made some headway in determining theoretically how the different abatement technologies and disinclinations toward pollution of sovereign countries might impact the identities of the countries that contribute the most to world pollution. All else equal, the countries which were most efficient at preventing or abating pollution specialized in the production of the pollution-intensive good. However, there was ample room for interference from countries that were experiencing extreme disutilities from pollution in one direction or another. In an effort to conform as closely as possible to a model by Copeland

and Taylor (2001), I sacrificed much analytical breadth. Ultimately, the inability to pin down a general equilibrium price meant that identifying which countries would contribute to global pollution specifically was beyond my reach. In the future, I would use a simpler but (I would hope) more general model that maintains some of this model's more essential features and answers my specific questions more directly.

4 Appendix

4.1 Deriving the Production Technologies

Copeland and Taylor (2001) demonstrate how to transform pollution from a bi-product of production into an input, starting with the dual production technology

$$Y = H(K^y, L^y);$$

$$X = (1 - \theta)F(K^x, L^x);$$

$$Z = \phi(\theta)F(K^x, L^x).$$

$\theta \in [0, 1]$ is a choice variable embodying the idea that the firm can only abate its emissions by reducing its production of the pollution-intensive good by a certain proportion. Meanwhile, whatever proportion of X that it does not devote to abatement will contribute to pollution Z in a manner scaled by the function $\phi(\theta)$. Defining $\phi(\theta) \equiv (1 - \theta)^{1/\alpha}$, the two authors show algebraically that the production technologies can be rewritten to exclude θ as just

$$Y = H(K^y, L^y);$$

$$X = Z^\alpha F(K^x, L^x)^{1-\alpha},$$

where Z is the stand-in choice variable for θ . Due to the constraint that $\theta \leq 1$, Z must be weakly less than X . Although it is a closed functional form, $\phi(\theta)$ grants each country its own abatement parameter, as captured by $\alpha \in (0, 1)$. The form of $\phi(\theta)$ implies that for any given value of θ , as α increases, pollution output will increase. Lower levels of α therefore correspond to more efficient abatement technologies. My model is more restrictive than Copeland and Taylor's because it assumes that $H(\cdot) = F(\cdot)$.

Conceptually and analytically, their construction incorporates a wonderful insight. The transformation from two co-dependent production functions to just one not only lets a person think more systematically about the firm's decision of whether to abate or not, but it also captures the notion that environmental health is in fact a resource that we must actively forfeit in our pursuit of greater and greater levels of production. Changes in F and ϕ_i are able to reverse the extent to which production creates pollution, so it is an error to assume that continual GDP growth need necessarily involve a more rapid exhaustion of the world's natural resources. If the world's collective abatement technologies manage to improve at a faster rate than output increases, then world output can increase coincident with aggregate decreases in pollution emissions. In the future, a dynamic model might allow such a phenomenon to be better analyzed.

4.2 Income Maximization

To help with determining the optimal level of Z_i , I employ the well-known result that profit-maximizing firms maximize the value of national income in a competitive economy. The firms take K_i , L_i , Z_i , α_i , and p as given. Meanwhile, they choose K_i^x , K_i^y , L_i^x , and L_i^y to maximize $pX_i + Y_i$ subject to equations (4)-(8). The optimality conditions for the problem are:

$$\frac{K_i^x}{L_i^x} = \frac{K_i^y}{L_i^y},$$

$$[p(1 - \alpha_i)]^{1/\alpha_i} Z_i = F(K_i^x, L_i^y),$$

and equations (6)-(8). Defining $a_i \equiv [p(1 - \alpha_i)]^{1/\alpha_i}$ and $A_i \equiv [p(1 - \alpha_i)]^{1/\alpha_i} F(K_i, L_i)^{-1}$ (so that $A_i F(K_i, L_i) = a_i$), and assuming for the moment that equation (8) is not binding, the solutions for the industries' inputs are

$$K_i^x = A_i Z_i K_i;$$

$$K_i^y = K_i(1 - A_i Z_i);$$

$$L_i^x = A_i Z_i L_i;$$

$$L_i^y = L_i(1 - A_i Z_i),$$

which when plugged into equations (4) and (5) yield outputs of

$$X_i = Z_i a_i^{1-\alpha_i};$$

$$Y_i = F(K_i, L_i) - a_i Z_i.$$

The maximal income when equation (8) does not bind is therefore

$$I_i = F(K_i, L_i) + Z_i \tilde{a}_i,$$

where $\tilde{a}_i \equiv \left(\frac{\alpha_i}{1-\alpha_i}\right) [p(1 - \alpha_i)]^{1/\alpha_i}$.

To complete the problem, suppose that equation (8) is binding, so that the optimal output of X_i derived above would exceed Z_i , contradicting equation (8). According to the equations, that contingency can only occur if $\alpha_i > \frac{p-1}{p}$. In that case, the best that the firms can do is to produce $X_i = Z_i$, which when combined with equation (5) would imply that $F(K_i^x, L_i^x) = Z_i$. The income maximization problem for such countries as have high abatement technologies is thus to choose their inputs to maximize $pZ_i + Y_i$ subject to $F(K_i^x, L_i^x) = Z_i$ along with

equations (4), (6), and (7). The solutions for the inputs in that case are

$$K_i^x = Z_i K_i F(K_i, L_i)^{-1};$$

$$K_i^y = K_i [1 - Z_i F(K_i, L_i)^{-1}];$$

$$L_i^x = Z_i L_i F(K_i, L_i)^{-1};$$

$$L_i^y = L_i [1 - Z_i F(K_i, L_i)^{-1}],$$

The resulting outputs are

$$X_i = Z_i;$$

$$Y_i = F(K_i, L_i) - Z_i,$$

and the maximum national income that satisfies equations (4)-(8) is

$$I_i = F(K_i, L_i) + (p - 1)Z_i.$$

Combining the two cases is what ultimately leads to equation (14).

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